Math 10B with Professor Stankova
Worksheet, Discussion #23; Tuesday, 4/17/2018
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1 Integrating Irreducible Quadratics

1.1 Concepts

1. For partial fractions, we often end up with something of the form $\frac{Ax+B}{x^2+bx+c}$ where the bottom is an irreducible polynomial, one that does not have real roots, and we need to integrate it. We will always end up with a $\ln x^2 + bx + c$ term and an arctan term. In order to split this, let $u = x^2 + bx + c$ so $du = 2x + b$ and write $Ax + B = Cdu + D$. Then you need to be able to complete the square and use the fact that

$$\int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \arctan(x/a) + C.$$

1.2 Example

2. Find $\int \frac{2x + 3}{4x^2 + 4x + 5} dx$.

1.3 Problems

3. Find $\int \frac{4x + 1}{x^2 + 4x + 8} dx$.

4. Find $\int \frac{3x + 4}{x^2 - 2x + 10} dx$.

1.4 Extra Problems

5. Find $\int \frac{x - 3}{x^2 - 4x + 5} dx$.

6. Find $\int \frac{5-x}{x^2-x+1} dx$.

2 Determinants and Inverses

2.1 Concepts

7. There are 2 ways to calculate the determinant of a 3×3 matrix $\begin{pmatrix} b_1 & b_2 & b_3 \\ b_4 & b_5 & b_6 \\ b_7 & b_8 & b_9 \end{pmatrix}$. The first is expansion along the first row to get the determinant is $b_1\begin{vmatrix} b_5 & b_6 \\ b_8 & b_9 \end{vmatrix} - b_2\begin{vmatrix} b_4 & b_6 \\ b_7 & b_9 \end{vmatrix} + b_3\begin{vmatrix} b_4 & b_5 \\ b_7 & b_8 \end{vmatrix}$. The other is to use the diagonal method to get $b_1b_5b_9 + b_2b_6b_7 + b_3b_4b_8 - b_1b_6b_8 - b_2b_4b_9 - b_3b_5b_7$.

   We can determine the number of solutions to an equation $A\vec{x} = \vec{b}$ by the determinant of A and that is given below.

$\det(A)$	$\neq 0$	$= 0$
Number of Solutions	1	0 or ∞

2.2 Examples

8. Find the determinant of $\begin{vmatrix} 3 & 2 & 3 \\ -1 & 5 & -3 \\ 7 & -1 & -1 \end{vmatrix}$.

2.3 Problems

9. True False It is possible for $A\vec{x} = \vec{0}$ to have no solutions.

10. True False If we know that $A\vec{x} = \vec{b}$ has no solutions, then we know what $\det(A)$ is.

11. Let $A = \begin{pmatrix} 2 & 4 & -1 \\ 2 & 2 & 5 \\ 6 & 8 & 9 \end{pmatrix}$. What is $\det(A)$? How many solutions that $A\vec{x} = \vec{0}$ have?

12. Let $A = \begin{pmatrix} 0 & 1 & 0 \\ 5 & 1 & 3 \\ 2 & 2 & 4 \end{pmatrix}$. What is $\det(A)$? How many solutions does $A\vec{x} = \vec{0}$ have?

13. Find the solution to $x + 2y = 3$ and $4x + 5y = 6$ using matrix vector form.

14. Write $x + y = 10, y + z = 5, x + z = -1$ in matrix vector form. How many solutions does it have?